WORKING PAPER | ISSUE 09/2021 | 14 JUNE 2021

STABILITY OF COLLUSION AND QUALITY DIFFERENTIATION: A NASH BARGAINING APPROACH

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How do incentives to collude depend on how asymmetric firms are? In many markets product quality is an important parameter that determines firms' market strategies. We study collusion in a quality-differentiated duopoly and we adopt a Nash bargaining approach to compute the collusive equilibrium and assess its stability. We derive collusive and deviation strategies as continuous functions of quality asymmetry. We obtain novel and surprising results. Stability of collusion is associated with quality differentiation in a non-monotonic way. For low levels of differentiation, an increase in quality difference makes collusion less stable. The opposite holds for high levels of differentiation. Also, while low quality firms are more likely to leave the cartel for small quality differences, high quality firms determine cartel stability when the quality difference is suffciently high. Our results have implications for empirical research, and antitrust enforcement.

Keywords: Collusion, Vertical dierentiation, Nash bargaining
JEL Classications: D43, L13, L40, K21

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Recommended citation:
Athanasopoulos, T., B. Dindaroglu and G. Petropoulos (2021) 'Stability of collusion and quality differentiation: a Nash bargaining approach', *Working Paper* 09/2021, Bruegel

1 Introduction

The relationship between firms' asymmetries and collusive behavior has been at the center of attention for antitrust practitioners as well as strategy theorists. In this paper, we investigate cartel stability in quality differentiated industries.

Many detected cartels refer to industries that exhibit market-share asymmetries[1], which are often due to vertical differentiation in product performance, brand image, or reputation. Quality is an important parameter which plays an important role in market decisions.[2] In digital and technology markets where production costs are falling[3] and big data analytics are extensively used, cost asymmetry becomes less relevant, and quality differentiation emerges as one of the important parameters for defining market strategies.

Two important questions relate to how the degree of quality differentiation affects the stability of cartels, and which firm - the innovative leader or a technological laggard - is more likely to abandon the collusive agreement.

The scarce literature on the topic (Häckner, 1994; Symeonidis, 1999; Bos and Marini, 2019, and; Ecchia and Lambertini, 1997) has investigated these questions by adopting a static joint profit maximization approach and imposing further ad hoc assumptions so that the solution is implementable (i.e., full market coverage, fixed market shares under collusion and competition). It concludes that quality asymmetry and stability of collusion have a monotonic relationship.

Static joint profit maximization is not an appropriate method for computing collusive agreements in many settings, especially under asymmetries and in the absence of inter-firm payments (Bain, 1948; Harrington, 1991). This is because it allocates production towards the most efficient firm without considering the dynamic stability of collusion, which depends on how this allocation affects the incentives of less efficient firms to participate. If the resulting allocation does not provide sufficient incentives for the latter to collude, it is not implementable.

[1]Ganslandt et al. (2012) report that in 43 cartel cases investigated by the European Commission between 2002 and 2007, the size of the second-largest firm was on average 70% of the size of the largest firm. Davies and Lyons (1996) find similar results for earlier years in the EU.

[2]In the Coty case (see Press Release No. 132/17 Luxembourg, 6 December 2017, Judgment in Case C-230/16 Coty Germany GmbH v. Parfumerie Akzente GmbH), the European Court of Justice concluded that market competition is multidimensional in online commerce and apart from the price component there are other relevant dimensions such as product quality and brand image.

[3]OECD (2015) observes declining costs along the data value chain which shift the attention from cost considerations to data-induced quality aspects in (online) commerce. The rise of cloud computing also contributed to the fall in production costs in a symmetric way, even if this fall is still difficult to be accurately captured by the official statistics (Byrne, Corrado and Sichel, 2018).

Given the trade-off between static joint profit maximization and dynamic stability when firms are asymmetric, a more appropriate method to study collusion is the Nash bargaining approach (Nash, 1950). Nash bargaining allows us to focus on the set of implementable subgame perfect Nash equilibrium collusive strategies. When the set of subgame perfect equilibria is large, as it is often the case in repeated game settings, it is natural to consider the firms to engage in bargaining over the set of potential outcomes (Harrington, 1991).

We adopt an infinite time horizon and we consider two firms with different quality levels (which we call the leader with high quality and the follower with low quality) that either compete in prices or collude, without any inter-firm payments. We analytically derive the Nash bargaining solution that jointly determines the level and the division of collusive profits.[4] Nash bargaining allows for the endogenous derivation of implementable collusive prices weighting both static profits and dynamic incentives for collusion, without having to rely on additional assumptions that may be difficult to justify.[5]

Assessing the stability of the derived collusive agreement requires to specify the optimal punishment mechanism for potential deviators. We show that in our case grim trigger strategies (Friedman, 1971) constitute the optimal punishment: The deviator enjoys a period of a deviation profit followed by (Nash equilibrium) price competition for all the remaining periods.

By constructing a specific parameter that denotes the degree of quality asymmetry we can, in turn, express competitive, collusive, and deviation strategies as functions of quality asymmetry. Our approach allows us to measure stability as a continuous function of quality difference. We show that this approach brings new insights that unravel how firms strategically respond to changes in the degree of quality asymmetry.

We find that the stability of collusion is related to the degree of quality differentiation in a non-monotonic way. For low levels of quality differentiation, an increase in quality asymmetry leads to less stable collusive agreements. But, the opposite holds for high degrees of quality differentiation.

[4] Alternatively, it is possible to consider the two stage approach (static joint profit maximization, followed by Nash bargaining) used by Schmalensee (1987) to compute the collusive equilibrium under cost asymmetry. The first stage is also illustrated in the exercise 6.1 of Tirole (1988). With the help of software packages we can derive the analytical solution of this two-stage problem under quality asymmetry. However, relying on Nash bargaining alone leads to identical results with this two stage approach. Since inter-firm payments are not allowed, it is the Nash bargaining problem that fully characterizes the collusive equilibrium. Nash bargaining, by definition, gives a solution at the Pareto frontier.

[5] Both Symeonidis (1999) and Häckner (1994) state that they resort to static profit maximization as an ad hoc way to compute the collusive equilibrium, as they were unable to derive the Nash Bargaining solution due to the involved computational difficulties.

We also show that it is the follower (leader) who has higher incentives to deviate from the collusive agreement if the quality difference between the two firms' goods is relatively low (high).

As the quality difference increases, the collusive price of the leader and the follower diverges. As more consumers will prefer the high-quality product, the collusive agreement adjusts the two prices so that the follower will keep its consumer base and will be able to derive sufficient profits so that it still wants to participate in the collusive agreement. Despite this price adjustment, collusive profits also diverge with vertical differentiation. They are reallocated from the follower to the leader.

Moreover, both firms have profitable deviations that attract the rival's consumer base in the deviation period. These one-period deviations are more profitable (in comparison to the equilibrium collusive profit) if differentiation is low because consumers are more tempted to switch to the deviator. This effect is stronger for the follower since deviation gives this firm access to higher-valuation consumers that are served by the leader under collusion. In contrast, for large quality differences, the follower finds it more difficult to capture additional consumers in the deviation period since more consumers prefer to consume the high quality good rather than switching, even though the follower's price is lower. At the same time, the one-period benefits from deviation relative to collusion decrease for the leader, as it serves all high valuation consumers through the collusive mechanism. Deviation to capture the low valuation consumers served by the follower is a less attractive option.

In fact, for each firm, both one-period deviation profits and competitive profits (which determine the strength of the punishment after deviation) get closer to collusive profits with quality asymmetry. Hence, as vertical differentiation increases we observe two countervailing effects: one-shot deviations become less attractive, but, at the same time, the punishment following the deviation is less severe. For low degrees of differentiation, it is the latter effect that dominates. Therefore, collusion becomes less stable with quality asymmetry. For large degrees of differentiation, it is the former effect that dominates, and collusion becomes more stable as the quality difference rises.

To our knowledge, we are the first to report the non-monotonic relationship between the stability of collusion and the degree of asymmetry between participating firms in vertically differentiated industries. We are also able to identify the firm that has the greatest incentives to deviate from the collusive agreement as a function of the exact degree of quality differentiation. Our results show that the efficient firm does not necessarily have stronger incentives to deviate from a collusive agreement than a less efficient firm, as it has been found by the literature.

The Nash bargaining solution has already been implemented in the literature that deals with cost asymmetry (Schmalensee, 1987, Harrington, 1991, Miklòs-Thal, 2011). The main conclusion in these papers is that cost asymmetry hinders collusion and that it is the least efficient firm that has more incentives to deviate from the collusive equilibrium. But, quality differentiation, unlike cost asymmetry, directly affects consumer preferences, which we model explicitly. Under cost asymmetry, consumers have to choose among identical products and their product choices are driven only by the price level of each firm. In our model, quality is an additional parameter that affects consumer choice, hence the collusive equilibrium and deviation strategies.[6]

We undertake a numerical exercise in which the two firms do not only differ in product quality but also in the marginal cost of production. This exercise suggests that our results on the non-monotonicity of cartel stability are retained with the addition of different marginal costs, while all our results remain qualitatively the same for small cost differences.

We also consider the case where direct monetary transfers are feasible as an extension.[7] For sufficiently high degrees of differentiation, we find that collusion is more stable when inter-firm payments are not feasible. This contradicts the conventional wisdom that collusion with side-payments leads to more stable collusive structures.

While side-payments require explicit coordination, our results when inter-firm payments are absent can be the outcome of either tacit or explicit coordination.[8] We use the terms cartel and collusion interchangeably throughout the paper.

The rest of the paper is organized as follows. We introduce our model in Section 2, and study the competitive equilibrium in Section 3. Section 4 characterizes the collusive equilibrium and states our main results. We study a model incorporating different marginal costs for different quality levels in Section 5. Section 6 studies the implications of the availability of side payments,

[6] Interestingly, we show that when firms compete in prices, quality asymmetry in our framework generates different optimal punishment mechanism than the one we expect under price competition with cost asymmetry (Miklòs-Thal, 2011).

[7] There is evidence that some cartels have used side payments. For example, as Pesendorfer (2000) reports, a bid-rigging scheme in Florida used side payments in the provision of school milk and dairies to compensate cartel members for refraining from bidding. Asker (2010) reports another cartel formed in stamp auctions in New York, where side payments were used for a similar reason. Probably, a more prominent example is the case of vitamin cartels (Igami and Sugaya, 2018): The heads of the vitamin divisions of big pharmaceutical companies agreed to freeze market shares at pre-determined levels and split sales according to these quotas. Side payments were arranged in the form of compensating sales from under-quota members to those who exceeded their quotas.

[8] Melkonyan et al. (2018) illustrate how firms can solve a (virtual) bargaining problem and collude tacitly.

and Section 7 concludes.

2 The Model

There are two firms, the leader (L) and the follower (F), interacting repeatedly in the same market over an infinite, discrete-time horizon. The stage game models a vertically differentiated industry setting in the tradition of Shaked and Sutton (1982). Each firm supplies a single product whose quality is given by q_i, $i = L, F$, with $q_L \geq q_F$. Our primary interest is in cases with $q_L > q_F$, while we briefly study the symmetric case with $q_L = q_F \equiv q_0$. We assume quality levels to be exogenously given. Firms simultaneously choose prices p_L and p_F to maximize the discounted sum of period profits Π_L and Π_F. The marginal costs of production for all products are normalized to zero[9], and firms have a common discount factor $\delta \in (0, 1)$. We denote the degree of differentiation by $k \equiv q_L/q_F$.

There is a continuum of heterogeneous consumers who differ in their valuations θ for product quality, where θ is uniformly distributed in the interval [0,1]. Each consumer has unit demand and obtains net utility

$$U(\theta) = \begin{cases} \theta q_i - p_i & \text{when buying from firm } i \\ 0 & \text{when not buying.} \end{cases} \tag{1}$$

Customers observe qualities and prices before making their purchasing decisions.

3 Competition

We begin by analyzing competitive prices and profits. If firms are symmetric ($q_L = q_F \equiv q_0$), Bertrand competition leads to marginal cost pricing, so that $p_L = p_F \equiv p_0^* = 0$ and $\Pi_L = \Pi_F \equiv \Pi_0^* = 0$.

It is straightforward to characterize the equilibrium of the stage game under asymmetry. Given qualities $q_L > q_F$ and prices $p_L > p_F$, there is an indifferent consumer situated at $\hat{\theta}$, given by

[9]We consider positive and distinct marginal costs in Section 5.

$\hat{\theta}q_L - p_L = \hat{\theta}q_F - p_F$, so that

$$\hat{\theta} = \frac{p_L - p_F}{q_L - q_F}. \tag{2}$$

Consumers with $\theta > \hat{\theta}$ buy from L as long as $\theta > p_L/q_L \equiv \theta_L$, and those with $\theta < \hat{\theta}$ buy from F as long as $\theta > p_F/q_F \equiv \theta_F$. Hence, the demand for L is given by $\max\{0, 1 - \max\{\theta_L, \hat{\theta}\}\}$, and the demand for F is $\max\{\min\{\hat{\theta}, 1\} - \theta_F, 0\}$. Assuming that prices satisfy $1 > \hat{\theta} \geq \theta_L \geq \theta_F > 0$, profits can be written as $\Pi_L = p_L(1 - \hat{\theta})$ and $\Pi_F = p_F(\hat{\theta} - \theta_F)$. Firms determine prices by maximizing profits, which leads to best response functions

$$p_L^{br} = \frac{p_F}{2} + \frac{q_F(k-1)}{2}, \qquad p_F^{br} = \frac{p_L}{2k}. \tag{3}$$

Solving the best response functions jointly, we get Nash equilibrium prices

$$p_L^* = \frac{2q_L(q_L - q_F)}{4q_L - q_F}, \qquad p_F^* = \frac{q_F(q_L - q_F)}{4q_L - q_F}, \tag{4}$$

and profits

$$\Pi_L^* = \frac{4q_L^2(q_L - q_F)}{(4q_L - q_F)^2}, \qquad \Pi_F^* = \frac{q_L q_F(q_L - q_F)}{(4q_L - q_F)^2}. \tag{5}$$

Note that the indifferent consumer in equilibrium is given by $\hat{\theta}^* = (2q_L - q_F)/(4q_L - q_F)$ and that the assumption $1 > \hat{\theta}^* > \theta_L^* > \theta_F^* > 0$ is satisfied in equilibrium.

4 Collusion

We consider collusion in prices when side payments between firms are prohibited or infeasible. By applying the Folk theorem, any collusive outcome is sustainable if there is an infinite time horizon, the discount factor is sufficiently high, and there is an efficient punishment mechanism for deviators. The collusive agreement typically cannot be enforced through legal instruments. Hence, we need to rely on the concept of subgame perfect Nash equilibrium in an infinitely repeated game setting where there is an underlying one period game with one or more Nash equilibria. Firms collude until one of them deviates, after which a grim trigger punishment phase occurs: In each period of the punishment phase, firms earn their competitive profits.

The sustainability of collusion requires

$$\Pi_i^c \geq (1-\delta)\Pi_i^d + \delta \Pi_i^*, \tag{6}$$

for each $i = L, F$, where the superscript (*) denotes the punishment phase, (c) denotes collusion, and (d) denotes deviation. Condition (6) implies that firm i does not deviate as long as

$$\delta \geq \hat{\delta}_i = \frac{\Pi_i^d - \Pi_i^c}{\Pi_i^d - \Pi_i^*}, \tag{7}$$

where $\hat{\delta}_i$ is a firm-specific threshold discount factor measuring the incentives of firm $i = L, F$ to deviate from the collusive agreement. Hence, the stability of collusion is determined by the discount factor $\hat{\delta} \equiv \max\{\hat{\delta}_F, \hat{\delta}_L\}$.

Proposition 1 shows that in our framework the grim trigger punishment (Friedman, 1971) is the optimal punishment mechanism in the sense of Abreu (1986, 1988), and therefore dominates any form of stick-and-carrot punishment.

Proposition 1. *The optimal punishment mechanism is the grim trigger punishment. Following the deviation, firms revert to the static Nash equilibrium for all the subsequent periods.*

Proof. We define as the optimal mechanism, the one that i) minimizes the expected payoff of the deviator; 2) it is credible such that the payoff of the non-deviator in the punishment phase is sufficiently high to implement that punishment. It suffices to show that there cannot be a more severe punishment for the deviator which is at the same time credible for the non-deviator. Let us assume that the leader deviates. The payoff of the leader and the follower under the grim trigger strategies, in the punishment phase, will be: $\frac{\Pi_L^*}{1-\delta}$ and $\frac{\Pi_F^*}{1-\delta}$, respectively.

Following Abreu (1986) a natural candidate mechanism will be the one which punishes harshly the deviator for the first τ periods of the punishment phase (the stick). Given expression (3), the most harsh punishment for the leader will be the follower to set $p_F = 0$ for the first τ periods. Then, for each period $t \leq \tau$, the leader gets payoff $\frac{q_F(k-1)}{4}$ which is smaller than Π_L^*. For $t > \tau$, let the follower charge price $p_F^o > 0$. This mechanism can be optimal only if the following two conditions are satisfied:

$$\frac{q_F(k-1)}{4}(1-\delta^\tau)+\delta^\tau\Pi_L(p_L^{br},p_F^o)<\Pi_L^*, \tag{8}$$

and

$$\delta^\tau\Pi_F(p_L^{br},p_F^o)\geq\Pi_F^*. \tag{9}$$

It is easy to see that there is no price p_F^o that satisfies both conditions simultaneously for any value of τ. From (3) we know that $\Pi_L(p_L^{br},p_F^o)$ is monotonically increasing in p_F^o. Hence, the condition (8) takes the minimum value for the lowest possible p_F^o for which condition (9) is satisfied with equality. At that minimum value, (8) is still violated, since the leader has higher payoff than in the grim trigger strategy. The proof follows analogous steps in case we consider less harsh punishments in which $p_F\in(0,p_F^*)$ in the first τ periods. Same logic and results apply in the case the follower is the deviator. $\square$

4.1 Symmetric benchmark

Collusion under symmetry ($q_L=q_F\equiv q_0$) is straightforward to characterize. Setting prices to p_0 leads to demand $1-p_0/q_0$ and profits $\Pi_i=p_0(1-p_0/q_0)$. Profit maximization gives $p_0^c=q_0/2$ and total profits $\Pi_0^c=q_0/4$, which are shared equally to give individual profits $q_0/8$ to each firm. The optimal deviation strategy is to slightly undercut the opponent and obtain monopoly profits $\Pi_0^d=q_0/4$ for one period. Then, the joint profit maximization gives the threshold discount factor $\hat{\delta}_0=1/2$.

4.2 Collusive equilibrium

In asymmetric environments, collusion requires specifying an agreement as to how collusive profits will be allocated among players. We consider an agreement whereby the joint cartel decision emerges from bilateral bargaining, where the disagreement point is the competitive profit allocation. This

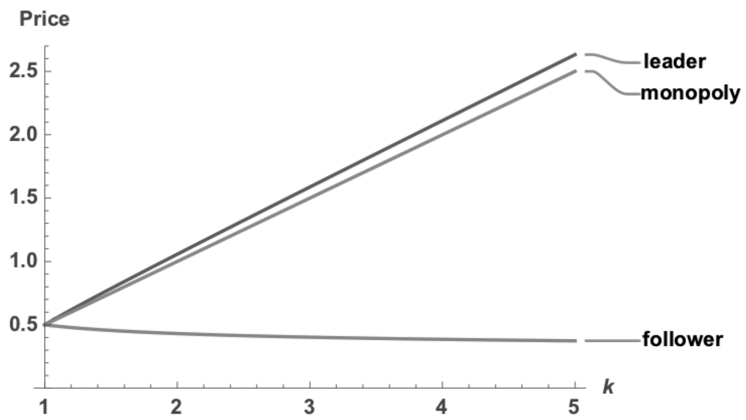


Figure 1: Collusive prices of leader and follower and monopoly price of leader as functions of the quality differentiation (k).

leads to the following Nash bargaining sharing rule:

$$\begin{aligned}\max_{p_L^c, p_F^c} &\{(\Pi_L^c - \Pi_L^*)(\Pi_F^c - \Pi_F^*)\} \\ s.t.\ &\Pi_L^c > \Pi_L^*,\ \ \Pi_F^c > \Pi_F^*,\end{aligned} \tag{10}$$

where

$$\Pi_L^c = p_L^c\left(1 - \frac{p_L^c - p_F^c}{q_L - q_F}\right), \qquad \Pi_F^c = p_F^c\left(\frac{p_L^c - p_F^c}{q_L - q_F} - \frac{p_F^c}{q_F}\right), \tag{11}$$

and p_L^c and p_F^c denote equilibrium collusive prices.

The bargaining problem in (10) leads to analytical solutions for collusive prices p_L^c and p_F^c.[10] The marginal consumers $\hat{\theta}^c$, θ_L^c and θ_F^c that determine demand functions are calculated using collusive prices, and satisfy $\hat{\theta}^c > \theta_L^c > \theta_F^c$. Figure 1 depicts $\frac{p_L^c}{q_F}$ and $\frac{p_F^c}{q_F}$ as well as $\frac{p_L^m}{q_F}$, where p_L^m is the monopoly price of the leader.[11]

The leader's collusive price is increasing in quality differentiation k while the respective price for the follower is decreasing. Thus, as the quality advantage increases, so does the equilibrium price differential. A higher price differential allows the follower to keep its base of consumers despite the increased quality asymmetry. As a result, the follower retains its collusive profit at a level that makes the collusive equilibrium sustainable for a range of the common discount factor δ. An

[10]Expressions are too lengthy to be reported here. The supplementary Mathematica file includes all calculations.
[11]Price p_L^m is computed from the first order condition of the maximization problem $\max_{p_L}\{p_L(1 - \frac{p_L}{q_F k})\}$.

interesting feature of the collusive equilibrium is that the leader charges a price that exceeds its monopoly price. The leader is willing to forgo a part of the monopoly profit by charging a higher price so that the follower has sufficient incentives to participate in the collusive equilibrium without deviating. The difference between the leader's collusive and monopoly price is increasing in k.

4.3 Deviation strategies

The optimal deviation strategy for each firm $i = L, F$ is to select the price that maximizes its profits given the rival firm's collusive price p_j^c, where $j = L, F$ and $j \neq i$. The deviator's best response to the other firm playing its collusive equilibrium strategy could potentially be an interior price choice - coming from the first-order conditions of its profit maximization problem - or a price that could force the competitor to have zero demand. This leads to:

$$p_L^d = \begin{cases} \frac{p_F^c}{2} + \frac{q_F(k-1)}{2} & \text{if} \quad p_F^c \leq \frac{q_F(k-1)}{2k-1}, \\ kp_F^c & \text{if} \quad \frac{q_F(k-1)}{2k-1} < p_F^c, \end{cases} \tag{12}$$

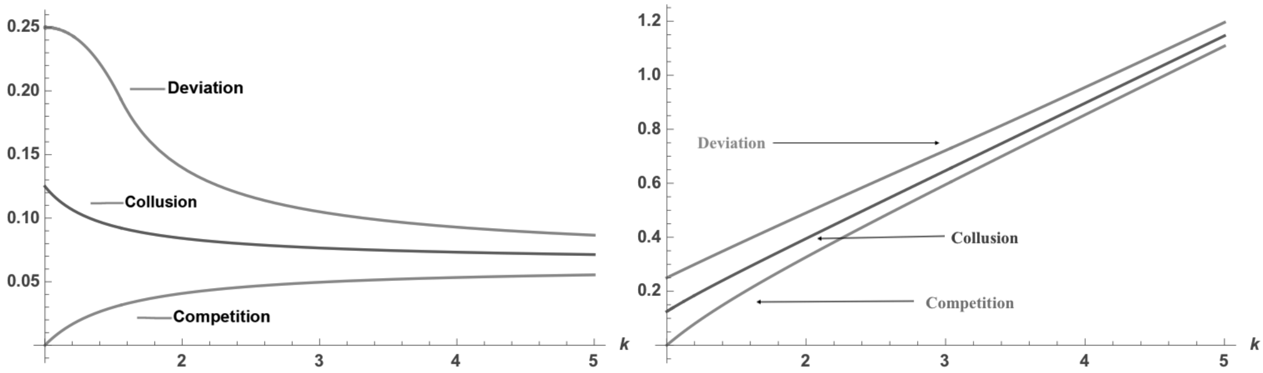
for the leader, and

$$p_F^d = \begin{cases} \frac{p_L^c}{2k} & \text{if} \quad p_L^c \leq q_F \frac{2k(k-1)}{2k-1}, \\ p_L^c - q_F(k-1) & \text{if} \quad q_F \frac{2k(k-1)}{2k-1} < p_L^c, \end{cases} \tag{13}$$

for the follower.[12]

When the deviation does not violate the constraint $\theta_F \leq \hat{\theta} \leq 1$, it is best for a firm to deviate according to the best response functions in (3), by maximizing own profits holding rival's price at its collusive level. These are stated by the first interval in the deviation functions above. However, price levels may be such that the deviating firm can push its rival to have zero demand in the deviation period. For the follower, this occurs if the best response function leads $\hat{\theta}^d$, the consumer that is indifferent between the two products, to be equal to θ_F^c, essentially leaving the leader with zero demand. In this price range, the follower undertakes a form of limit pricing to keep the leader's demand at zero and serve all consumers with $\theta \in [\theta_F, 1]$ in the deviation period. A similar, but slightly different strategy exists for the leader, whose limit pricing deviation leads to the binding

[12] Note that equilibrium collusive prices satisfy $p_F^c \leq \frac{q_F}{2}$ and $p_L^c \leq q_F \frac{2k-1}{2k-1}$, $\forall k > 1$. We present the deviation strategies that may arise given the collusive equilibrium strategies. If we also include deviations off equilibrium paths in the analysis, there is a third deviation strategy for the leader (when follower's collusive price is greater than $\frac{q_F}{2}$) and the follower (when leader's collusive price greater than $\frac{2k-1}{2k-1}$) for which the deviator charges its monopoly price.



(a) (b)

Figure 2: Collusive, competitive and deviation profits for (a) the follower and (b) the leader as a function of k.

constraint $\hat{\theta} = \theta_F$, which effectively keeps F out of the market in the deviation period.

4.4 Stability of collusion

The collusive, competitive, and deviation profits for the leader and the follower are depicted in Figure 2. Collusive and one-shot deviation profits are monotonically increasing (decreasing) for the leader (follower) as quality differentiation (k) increases, while competitive profits of both firms are increasing functions of k. Note that when the degree of differentiation is low, both firms have profitable one-shot deviations that allow them to serve the other's customers during the deviation period. However, the resulting incentives for one-shot deviations diminish as differentiation is larger since stealing the rival's consumers becomes more difficult and costly as it requires a larger price cut. In contrast, competitive profits become more attractive for the colluding firms as k increases.

The threshold discount factors, which we call $\hat{\delta}_L^c$, $\hat{\delta}_F^c$, are calculated using (7). These are depicted in Figure 3 as functions of k. The stability of collusion is determined by $\hat{\delta}^c = \max\{\hat{\delta}_L^c, \hat{\delta}_F^c\}$.

The following two propositions summarize our main results:

Proposition 2. *For each of the two firms, the relationship between the threshold discount factor and quality differentiation (k) is an inverted-U. The follower has stronger incentives to deviate from the collusive agreement if* $1 < k < 1.869$*, while the leader has stronger incentives to deviate if* $k > 1.869$.

The incentives of both the leader and the follower to deviate from the collusive equilibrium follow

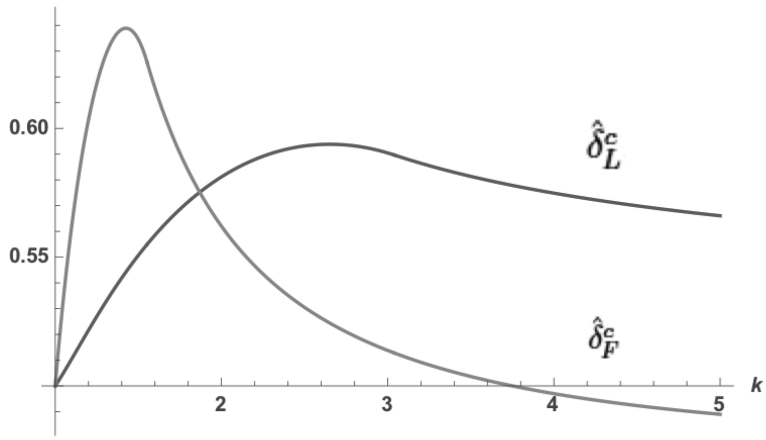


Figure 3: The critical discount factors of the leader (blue) and the follower (orange) as functions of quality differentiation (k).

an inverted-U pattern with k. The peak occurs at a lower value of k for the follower. For each firm, as k increases, while the one-period deviation becomes a relatively less attractive option, the punishment (competitive) payoff becomes relatively more attractive. For low-quality differences, it is the latter effect that dominates and collusion becomes less stable with k. For high-quality differences, it is the former effect that dominates and hence collusion becomes more stable.

Indeed, the difference between the one-period deviation profit and static collusive profit declines at a lower (higher) rate than the difference between static profits under collusion and competition for both firms when quality differentiation is low (high).

Furthermore, the firm that determines cartel stability depends on the degree of quality differentiation. More precisely, for lower values of the quality difference, the follower is more tempted to deviate from the collusive agreement compared to its counterpart. This is because, for low degrees of differentiation, deviation allows the low-quality firm to steal high-valuation consumers that are served by the leader under collusion. As the degree of differentiation rises, however, the follower finds it less attractive to deviate, as the leader's high-valuation customers optimally purchase the high-quality product even if the follower tries to lure them away with a lower price. This diminishing tendency of the follower to leave the cartel, as the degree of differentiation rises, makes the leader more prone to deviate since its punishment payoff is higher.

Looking at the overall picture, the non-monotonic relationship between firm's incentives to deviate from the cartel and the degree of vertical differentiation is naturally passed on to cartel stability. So, the relationship between cartel stability and k is non-monotonic as well:

Proposition 3. *There exist cutoffs* $\overline{k} = 1.426, \tilde{k} = 1.829, \hat{k} = 2.65$, *such that the cartel becomes (a) more stable with increased quality differentiation when* $\overline{k} < k < \tilde{k}$ *or* $k > \hat{k}$, *(b) less stable with vertical differentiation when* $k < \overline{k}$ *or* $\tilde{k} < k < \hat{k}$.

These results deviate from the literature in vertically differentiated industries, according to which i) there is a monotonic relationship between the quality asymmetry and collusion (i.e., Häckner, 1994; Symeonidis, 1999; Ecchia and Lambertini, 1997), and ii) a single firm has uniformly higher incentives to abandon the cartel: either the high-quality firm (Häckner, 1994) or the technological laggard (Symeonidis, 1999; Bos and Marini, 2019). Adopting a Nash bargaining approach that allows to determine endogenously the collusive equilibrium instead of computing this equilibrium in an ad hoc way provides new insights on firms' equilibrium strategies that have direct implications for incentives to collude.

5 Different marginal costs

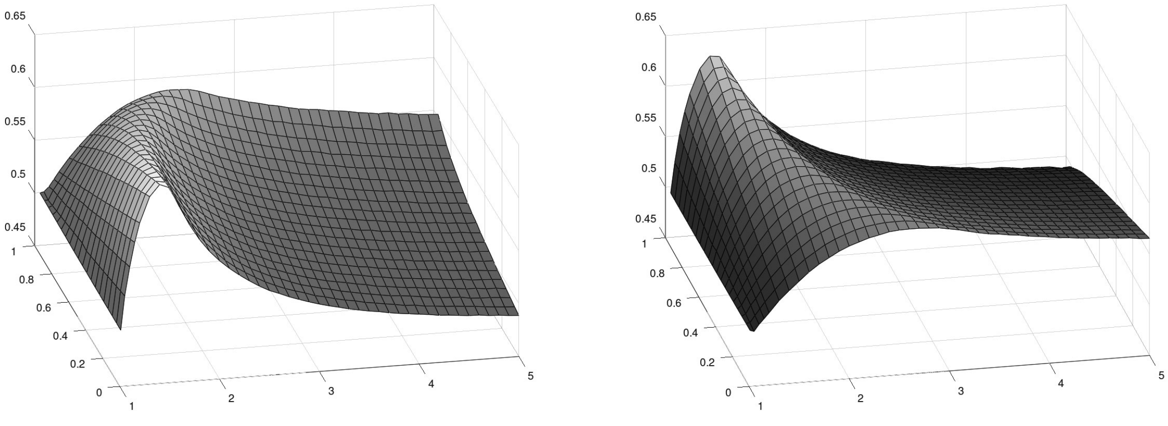
In this section, we incorporate non-zero marginal costs of production to our baseline model presented in the previous sections. We characterize the collusive equilibrium and its stability when the two firms have different marginal costs, denoted c_L (leader) and c_F (follower). It is natural to consider marginal costs of production to increase with product quality, hence to assume $c_F \leq c_L$. [13]

In the presence of non-zero marginal costs, the following two constraints need to be satisfied for competitive profits to be non-negative:

$$c_L \leq q_F(k-1)\frac{2k}{2k-1} + c_F\frac{k}{2k-1}, \qquad c_F \leq q_F\frac{k-1}{2k-1} + \frac{c_L}{2k-1}. \tag{14}$$

We study the effects of various cost configurations for each k under the constraints stated above. To illustrate the main results from this analysis, we present results on the stability of collusion with the following simplifications: we normalize c_F to zero, q_F to one, and use the cost specification $c_L = c(k-1)$ for the leader where c is a given constant. This allows the marginal cost of the leader

[13] The collusive equilibrium under different marginal costs is obtained numerically. Details of the model are not presented for space considerations. A detailed description of the model as well as the Octave/Matlab files that are used to generate solutions are available as supplementary files.



(a) (b)

Figure 4: Threshold discount factors of (a) the follower and (b) the leader as functions of k and c.

to increase with product quality and allows us to vary parameter c while respecting (14). The effect of increasing marginal cost differences is shown in Figure 4, which depicts threshold deviations for both firms as functions of k and c.

The results reveal that the inverted-U relationship between quality differentiation and cartel stability is retained for each firm as the cost difference increases. For small cost differences, the relative incentives to deviate for the leader and the follower are similar to our finding stated in Proposition 2: the follower has higher incentives to deviate for small k, while the leader has higher incentives to deviate for large k values.

This ranking of incentives is overturned for larger cost differences. As in our baseline model, individual incentives and the overall stability of collusion is determined by the interplay between two mechanisms; one relating to the desirability of deviation, and the other the threat of punishment. As c_L increases, so does the leader's collusive price, p_L^c. As a result, $\hat{\theta}^c$ increases and approaches one, hence the leader serves an increasingly smaller fraction of the highest-valuation consumers. Accordingly, deviation gives this firm access to a larger fraction of additional high-valuation consumers below its demand threshold, which renders deviation more attractive. At the same time, the relative value of deviation for the follower is diminished, since $\hat{\theta}^c$ is higher and the leader's consumers are more difficult to divert from consuming its product. As a result, the incentive structure that led to the threshold functions in Figure 3 is reversed. This deviation effect is stronger for small k values. On the other hand, an increase in c_L reduces (increases) the competitive profits of the leader (follower), hence makes the deterrence effect stronger (weaker) for this firm. Hence, the leader has weaker, while the follower has stronger incentives to deviate compared to our baseline model. This effect is more dominant for larger k values. Figure 4 exhibits the outcome of the combined effect: the reversal in incentives, as well as the effect of increasing cost differences for a given value of the quality differential.

The non-monotonic relationship between cartel stability and differentiation stated in Proposition 3 is qualitatively retained for each value of the marginal cost difference.

6 Collusion with side payments

We now consider the case in which intra-firm payments are feasible. Under such conditions, the colluding firms can attempt to maximize joint period profits and implement this solution using side payments. Unlike our baseline model, closed-form solutions for the collusive equilibrium and threshold discount factors are easy to obtain. The joint profit maximization problem of the cartel can be written as

$$\Pi^{sp} = \max_{p_F, p_L} \left\{ \Pi_L^{sp} + \Pi_F^{sp} \right\},$$

where $\Pi_L^{sp} = p_L^{sp} \left(1 - \frac{p_L^{sp} - p_F^{sp}}{q_L - q_F} \right)$ and $\Pi_F^{sp} = p_F^{sp} \left(\frac{p_L^{sp} - p_F^{sp}}{q_L - q_F} - \frac{p_F^{sp}}{q_F} \right)$. The first order conditions for p_L and p_F give

$$p_F^{sp} = \frac{q_F}{2}, \qquad p_L^{sp} = p_F^{sp} + \frac{q_L - q_F}{2} = \frac{q_L}{2}, \tag{15}$$

which also imply $\hat{\theta}^{sp} = \theta_L^{sp} = \theta_F^{sp} = 1/2$, and give total cartel profits equal to $\Pi^{sp} = q_L/4$. All sales are made by the leader. As a consequence, this outcome can only be implemented using side payments.

We again consider a Nash bargaining rule for the sharing of total collusive profits:

$$\max_{\Pi_L^{sp}, \Pi_F^{sp}} \left\{ (\Pi_L^{sp} - \Pi_L^*)(\Pi_F^{sp} - \Pi_F^*) \right\}$$
$$s.t. \; \Pi_L^{sp} + \Pi_F^{sp} = \frac{q_L}{4}, \tag{16}$$

which leads to equilibrium collusive profits

$$\Pi_L^{sp} = \frac{8q_L^2 - 5q_L q_F}{8(4q_L - q_F)}, \qquad \Pi_F^{sp} = \frac{3q_L q_F}{8(4q_L - q_F)}. \tag{17}$$

Note that the collusive participation constraints are satisfied for all q_L and q_F with $q_L > q_F > 0$. To implement the strategy, the leader makes all sales and pays an amount equal to Π_F^{sp} in (17) to the follower in each period.

It is easy to see that the leader's optimal deviation from the collusive agreement is to refuse to make the side payment to the follower, which gives the deviation profit $\Pi_L^{d,sp} = \Pi^{sp} = q_L/4$. The optimal deviation strategy of the follower is derived in an analogous way to the previous section. This leads to the two-part deviation profits

$$\Pi_F^{d,sp} = \begin{cases} \frac{q_L(2q_F - q_L)}{4q_F} & \text{if } 1 < k < \frac{3}{2}, \\ \frac{q_L q_F}{16(q_L - q_F)} & \text{if } \frac{3}{2} \leq k. \end{cases} \tag{18}$$

Note that $\Pi_L^{d,sp} > \Pi_L^{sp}$ for all $q_L > q_F > 0$. However, $\Pi_F^{d,sp} > \Pi_F^{sp}$ only if $k < \frac{5}{2}$. For higher differentiation with $k \geq \frac{5}{2}$, the follower never deviates from the collusive agreement.

The critical discount factors for both firms can then be computed using (7) as

$$\hat{\delta}_L^{sp} = \frac{3}{4}\left(1 - \frac{3}{1 + 8k}\right) \tag{19}$$

and

$$\hat{\delta}_F^{sp} = \begin{cases} 1 - \frac{4k+5}{12 - 42k + 80k^2 - 32k^3} & \text{if } 1 < k < \frac{3}{2}, \\ \frac{(4k-1)(5-2k)}{3(8k-5)} & \text{if } \frac{3}{2} \leq k, \end{cases} \tag{20}$$

respectively. Note that $\hat{\delta}_L^{sp}$ is strictly increasing in k; the leader finds deviation to be more attractive as differentiation between the two firms increases. However, $\hat{\delta}_F^{sp}$ exhibits an inverted-U relationship with k. There is a critical value, k_F^{cr} (numerical value 1.234) that maximizes the follower's incentives to deviate.

Let $\hat{\delta}^{sp} = \max\{\hat{\delta}_F^{sp}, \hat{\delta}_L^{sp}\}$. The follower has stronger incentives to deviate from the collusive agreement if $1 < k < \frac{5}{4}$ ($\hat{\delta}^{sp} = \hat{\delta}_F^{sp}$), while the leader is more likely to deviate from the cartel arrangement if $\frac{5}{4} < k$ ($\hat{\delta}^{sp} = \hat{\delta}_L^{sp}$).

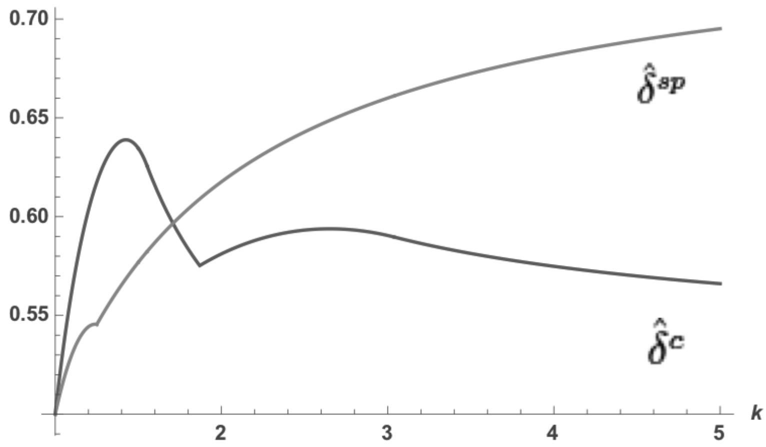


Figure 5: Critical discount factors for the stability of collusion with and without side payments as a function of quality differentiation (k).

Comparing the stability of collusion, $\hat{\delta}^c$ of our baseline model above with the side payments case, $\hat{\delta}^{sp}$ (Figure 5) we see that:

Proposition 4. *There is a cutoff value for quality asymmetry,* $k^* = 1.708$*, above which collusion is more stable in the absence of side payments.*

This indicates that side payments can lead to the destabilization of the collusive agreement for high levels of quality differentiation. This is because the leader has stronger incentives to deviate and capture monopoly profits by not providing the side payment to the follower. Higher degrees of differentiation also guarantee larger competitive profits in the punishment phase that will follow. In contrast, when side payments are not feasible, the leader's deviation can never be as profitable as in the side payments case. This is particularly true for high degrees of quality differentiation.

7 Conclusion

In this paper, we investigate cartel stability in a quality differentiated duopoly. We deviate from ad hoc assumptions that have been commonly used in the literature by relying instead on the Nash bargaining approach.

We find that the relationship between cartel stability and quality differentiation is non-monotonic.

In addition, a low quality (high quality) firm has higher incentives to deviate from the collusive agreement for low (high) degrees of differentiation between competitors. We also find that side payments can render collusion more stable *only if* product qualities in the industry are sufficiently close to one another.

Understanding the incentives to collude is important for organizing deterrence mechanisms that promote competition. In this respect, our model predictions shed light on the incentives of market leaders and followers to collude, in cases the quality of products and services is an important strategic variable (as in digital ecosystems and technology markets). In many instances, deterrence of collusive agreements relies on identifying potential whistle-blowers within the firms that only have weak incentives to collude.

Our approach and results have important implications for future research. The literature on the relationship between collusion and innovation largely deals with cost-reducing innovation. Our analysis paves the way for investigating the relationship between collusion and innovation when innovation improves a product in technological performance or in use-value. Extending our model to study the relationship between R&D competition and collusion on a learning curve (e.g., by adding a quality investment step in each firm's decision problem per period) is part of our current research efforts.

The computational difficulties introduced by the general market setting restricted our efforts to the case of a duopoly. The generalization of our model to an oligopoly with an arbitrary number of firms is also part of our ongoing research.

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